# Useful Formulas

**1. Design equations for attenuator (with the requirement $Z_I = R_L$):**
$K$ = voltage divider ratio = $V_{in}/V_{out}$, $R_L$ = load resistor.

- L attenuator

$$R_1 = \frac{K-1}{K} R_L \qquad R_2 = \frac{1}{K-1} R_L$$

- T attenuator

$$R_1 = \frac{K-1}{K+1} R_L \qquad R_2 = \frac{2K}{K^2-1} R_L$$

- π attenuator

$$R_1 = \frac{K^2-1}{2K} R_L \qquad R_2 = \frac{K+1}{K-1} R_L$$

**2. Important parameters for RLC resonance circuits:**

| Parameter | Series RLC network | Parallel RLC network |
|---|---|---|
| Input impedance | $R_s + j\omega L_s + \frac{1}{j\omega C_s}$ | $\left(\frac{1}{R_P} + \frac{1}{j\omega L_P} + j\omega C_P\right)^{-1}$ |
| Resonance frequency | $\omega_o = \frac{1}{\sqrt{L_s C_s}}$ | $\omega_o = \frac{1}{\sqrt{L_p C_p}}$ |
| Quality factor, Q | $Q_s = \frac{\omega_o L_s}{R_s} = \frac{1}{\omega_o R_s C_s}$ | $Q_p = \frac{R_P}{\omega_o L_p} = \omega_o R_P C_p$ |
| Bandwidth BW | $\frac{\omega_o}{Q_s}$ | $\frac{\omega_o}{Q_p}$ |

**3. Series to parallel and parallel to series reactive network transformation:**

$$R_s = \frac{R_p}{1+Q^2} \qquad X_s = \frac{X_p}{1+\frac{1}{Q^2}}$$

**4. L impedance matching network (when $R_L > R_S$):**

$$B = \frac{X_L \pm \sqrt{R_L / R_s}\sqrt{R_L^{\,2} + X_L^{\,2} - R_s R_L}}{R_L^{\,2} + X_L^{\,2}}\ ;\ X = \frac{1}{B} + \frac{X_L R_s}{R_L} - \frac{R_s}{B R_L}\ ;\ Q = \frac{|X|}{R_S}$$

**5. L impedance matching network (when $R_L < R_S$):**

$$B = \pm\frac{\sqrt{(R_s - R_L)/R_L}}{R_s}\ ;\ X = \pm\sqrt{R_L(R_s - R_L)} - X_L\ ;\ Q = R_S|B|$$

**6. π impedance matching network:**

$$L = L_1 + L_2 = \frac{R_I}{\omega_o}\left(\sqrt{R_S / R_I - 1} + \sqrt{R_L / R_I - 1}\right)$$

$$C_1 = \frac{1}{\omega_o R_S}\sqrt{\frac{R_S}{R_I} - 1}$$

$$C_2 = \frac{1}{\omega_o R_L}\sqrt{\frac{R_L}{R_I} - 1}$$

$$Q = \sqrt{\frac{R_S}{R_I} - 1} + \sqrt{\frac{R_L}{R_I} - 1}$$

**7. RF Class A transistor amplifier:**

- **Hybrid π model of BJT**

| | |
|---|---|
| $r_{bb'}$ | **The base-spreading resistance.** |
| $g_m$ | **The transconductance.** $g_m = \frac{dI_C}{dV_{BE}} = \frac{qI_C}{kT} \cong \frac{I_C\,(\text{in mA})}{26}$ <br> at $T$=25°, $q$= electronic charge, $1.602 \times 10^{-19}$C |
| $C_e$ | **The emitter capacitance**. |
| $r_{b'e}$ | **The input resistance.** <br> $r_{b'e} = \frac{dV_{B'E}}{dI_B} = \frac{h_{fe}}{g_m}$ |
| $r_{b'c}$ | **The collector to base resistance.** $r_{b'c} = h_{fe} r_{ce}$ |
| $C_C$ | **The collector capacitance**. |
| $r_{ce}$ | **The output resistance.** $r_{ce} \cong \frac{V_A}{I_C} = \frac{qV_A}{kTg_m}$ <br> where $V_A$ is known as the Early voltage and $I_C$ is the dc collector current. |

- **High frequency Class A amplifier parameters:**
- **Voltage gain:**

$$A_V = \frac{v_o}{v_{BE}} = -g_m R_o \frac{r_{b'e}}{r_{bb'} + r_{b'e}} \frac{1}{1 + sC_T \frac{r_{bb'} r_{b'e}}{r_{bb'} + r_{b'e}}} = \frac{K_1}{1 - \frac{s}{p_1}}$$

- **Effective voltage gain** (including the effect of source impedance $R_S$):

$$A_{Ve} = \frac{v_o}{V_S} = -g_m R_o \frac{r_{b'e}}{r_{bb'} + R_S + r_{b'e}} \frac{1}{1 + sC_T \frac{(R_S + r_{bb'}) r_{b'e}}{R_S + r_{bb'} + r_{b'e}}} = \frac{K_2}{1 - \frac{s}{p_2}}$$

where $C_T = C_e + C_M$ and $C_M$ = Miller capacitance

$$C_M = (1 + g_m R_o) C_C$$

- **Transition frequency**

$$f_T = \frac{1}{2\pi}\frac{g_m}{C_e + C_C}$$

- **Input impedance**

$$Z_I = r_{bb'} + \frac{r_{b'e}}{1 + s r_{b'e} C_T}$$

- **Maximum output voltage swing (for class A tuned amplifier)**

$V_{PP(max)} = 2(V_{CC} - V_E + V_{CE(sat)})$

**8. Phase-Locked Loop:**

$k_\phi$ = Phase detector gain, $k_A$ = Loop amplifier gain, F($s$) = Loop filter frequency response, $k_o$ = Voltage controlled oscillator gain.

- **Static phase error**

$$\theta_e = \frac{\Delta f}{k_o k_A k_\phi} = \frac{\Delta f}{k_L}$$

- **Hold-in range or lock range**

$$\Delta f_H = \theta_{e(\max)} |k_L|$$

- **Simple PLL or uncompensated PLL frequency response (**F($s$) = 1**)**

$$\frac{V_o(s)}{\theta_i(s)} = \frac{s k_\phi k_A}{s + k_V}$$

$$\frac{V_o(s)}{\Delta f_i(s)} = \frac{2\pi k_\phi k_A}{s + k_V} = \frac{k_V / k_o}{s + k_V}$$

- **Simple PLL or uncompensated PLL time domain response when there is a step change in input frequency**

$$V_o(t) = \frac{\Delta f}{k_o}\left(1 - e^{-k_V t}\right)$$

- **First order PLL or compensated PLL time domain response when there is a step change in input frequency**

$$V_o(t) = \frac{\Delta f}{k_o}\left[1 - \frac{e^{-\delta\omega_n t}}{\sqrt{1-\delta^2}}\sin\left(\omega_n t\sqrt{1-\delta^2} + \theta\right)\right]$$

$$\omega_n^{\,2} = k_V\left(\frac{1}{RC}\right)$$

$$\delta = \frac{1}{2\omega_n RC} = \frac{1}{2\sqrt{k_V RC}}$$

$$\theta = \tan^{-1}\sqrt{\frac{1}{\delta^2} - 1}$$