Chapter 2 Radiating Systems

2.1 ANTENNA FUNDAMENTALS

Antenna is the transition region between a guided wave propagating in a transmission line and an electromagnetic free space wave or vice-versa. Antennas are used to radiate and receive electromagnetic waves.

The structures of antenna can be a single straight wire or a conducting loop excited by a voltage source, and aperture at the end of a wave guide , or a complex array of radiating elements. A short linear conductor is called a *short dipole* or *Hertzian dipole*. The length L of dipole is very short compared to wave length, λ. In such antenna the current vanishes at the ends of the wire where charges must be accumulated.

Any linear antenna can be analyzed by treating it as a combination of large number of short dipoles connected in series.

2.2 RETARDED VECTOR POTENTIAL OF SHORT DIPOLE

Consider a very short wire carrying an a.c. current varying sinusoidally in time.

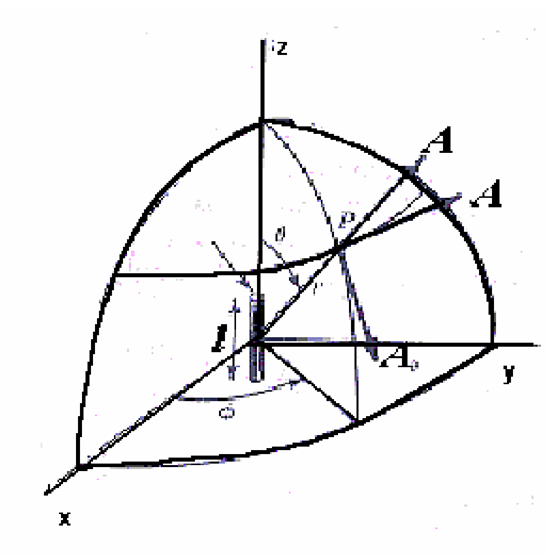


Figure 1 Retarded potential from small current element.

The instantaneous current can be expressed as

$$I = I_m \cos(\omega t) \qquad (1)$$

where

I_m is the maximum or peak current;

I is the current at any instant;

$\omega = 2\pi f$ is the angular frequency.

The vector electric potential expression represents the superposition of potentials due to various current elements (*I dl*), at any point *P,* at distance r from the element.

Now, introduce the concept of *retardation*. The electromagnetic waves have finite propagation times. This results in a time delay between the sources and potentials at a distance from the sources. The effect (potential) observed a distant point P from a given source at any instant t is due to a current flowing at an earlier time $\left(t - \frac{r}{c}\right)$.

The instantaneous current given by equation (1) is modified as

$$[I] = I_o cos\left[\omega\left(t - \frac{r}{c}\right)\right] \tag{2}$$

where

r is the distance between observation point and source;
c is the velocity of propagation;
[I] is the retarded current;
$(t - \frac{r}{c})$ is the retarded time .

The magnetic vector potential $\vec{A}$ is along z-direction having only z-component, A_Z retarded in time by "$\frac{r}{c}$" seconds.

Since $I = \int J\,ds \quad and \quad dv' = ds \cdot dl$

where ds is the cross-section area and dl is the length and $\beta = \frac{2\pi}{\lambda} = \frac{\omega}{c}$,

the equation for the retarded vector magnetic potential can be written as

$$A_z = \frac{\mu I_m l \cos(\omega t - \beta r)}{4\pi r} \tag{3}$$

2.3 Radiation Fields Of Elemental Dipole

The expressions of radiated electromagnetic fields from Hertzian dipole are found using the following procedure.

Get the vector potential function $\overline{A}$ and scalar potential V first. From the Maxwell's equations, in the medium with source, we can construct the non-homogeneous wave equations.

$$\nabla^2 \overline{A} - \mu\varepsilon \frac{\partial^2 \overline{A}}{\partial t^2} = -\mu\overline{J} \tag{4}$$

$$\nabla^2 V - \mu\varepsilon \frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\varepsilon} \tag{5}$$

The solutions to (4) and (5) are

$$\overline{A} = \frac{\mu}{4\pi}\int_{V'} \frac{\overline{J}e^{-jkR}}{R} dv' \tag{6}$$

and

$$V = \frac{1}{4\pi\varepsilon}\int_{V'} \frac{\rho e^{-jkR}}{R} dv' \tag{7}$$

where $k = \omega\sqrt{\mu\varepsilon} = 2\pi / \lambda$ is the wavenumber.

Both pairs of $(\rho, \bar{J})$ and $(V, \bar{A})$ can be related through the following expressions:

$$\nabla \cdot \bar{A} + j\omega\mu\varepsilon V = 0 \qquad (8)$$

and

$$\nabla \cdot \bar{J} = -j\omega\rho \qquad (9)$$

- Determine $\bar{A}$ from $\bar{J}$ using (6) -- [Integration]
- Find $\bar{H}$ using $\bar{H} = \frac{1}{\mu}\nabla \times \bar{A}$ -- [Differentiation]
- Obtain $\bar{E}$ using $\bar{E} = \frac{1}{j\omega\varepsilon}\nabla \times \bar{H}$ -- [Differentiation]

Consider an electric dipole of length l along the z-axis centred on the coordinate origin. In this case, the volume integral for vector potential reduces to one-dimensional integral, so that equation (6) becomes

$$A_z = \frac{\mu I_m l e^{i\omega(t - r/c)}}{4\pi r} \qquad (10)$$

It will be desirable to obtain *E* and *H* in polar coordinates.

The magnetic field intensity is obtained from the magnetic potential,

$$B = \nabla \times A = \mu H$$

and it is seen that the components of *A* are

$$A_\phi = 0, \frac{\partial}{\partial \phi} = 0, A_r = A_z \cos(\theta), and\, A_\theta = -A_z \sin(\theta).$$

(Due to spherical symmetry, the field is symmetrical. So, $\frac{\partial}{\partial \phi} = 0$)

Now from $(\nabla \times A)$ in its polar coordinate components and using equation (10), we yield

$$H_r = \frac{1}{\mu}(\nabla \times A)_r = 0$$

$$H_\theta = \frac{1}{\mu}(\nabla \times A)_\theta = 0$$

$$\begin{aligned} H_\phi &= \frac{1}{\mu}(\nabla \times A)_\phi = \frac{1}{r}\left[\frac{\partial}{\partial r}(A_\theta r - \frac{\partial A_r}{\partial \theta})\right] \\ &= \frac{1}{r}\left[\frac{\partial}{\partial r}(-A_z r \sin\theta) - \frac{\partial}{\partial \theta}(A_z \cos\theta)\right] \\ &= \frac{I_m l \sin(\theta) e^{j(\omega t - \beta r)}}{4\pi}\left[\frac{j\beta}{r} + \frac{1}{r^2}\right] \end{aligned} \qquad (11)$$

The components of **E** can be calculated using Maxwell's equation,

$$\nabla \times H = \varepsilon \frac{\partial E}{\partial t}.$$

Thus, the electric field components are

$$E_r = \frac{I_m l \cos(\theta) e^{j(\omega t - \beta r)}}{2\pi}\left[\frac{\eta}{r^2} + \frac{1}{j\omega\varepsilon r^3}\right]$$

$$E_\phi = 0$$

$$E_\theta = \frac{I_m l \sin(\theta) e^{j(\omega t - \beta r)}}{4\pi}\left[\frac{j\omega\mu}{r} + \frac{\eta}{r^2} + \frac{1}{j\omega\varepsilon r^3}\right] \qquad (12)$$

In the above equations, $\eta = \sqrt{\frac{\mu}{\varepsilon}}$ represents the intrinsic impedance of the medium ($\eta = 120\pi$ ohms, in free space).

The field of the short dipole has only three components namely E_r, E_θ, H_ϕ.

2.4 Induction (Near) Field and Radiation (Far) Field

Consider the expression for magnetic field intensity.

$$H_\phi = \frac{I_m l \sin(\theta) e^{j(\omega t - \beta r)}}{4\pi}\left[\frac{j\beta}{r} + \frac{1}{r^2}\right]$$

2.4.1 The Radiation Field

The first term varies inversely as distance (e.g. 1/r) and it is known as radiation field or far field. This field is of great significance at large distance.

2.4.2 The Induction Field

The second term varies inversely as square of the distance and it is known as near field or induction field. Induction field will be predominant at points close to the current element, where *r* is small.

Consider now the expressions for Electrical field intensity.

- Induction term contains; $1/r^2$ term.
- Radiation term contains, $1/r$ term.

The term varying inversely as a cube of distance ($1/r^3$), is called as electrostatic field, due to its similarity with components of an electrostatic dipole. (*It is important near the current element*).

We can consider that the space is divided into three regions.

2.4.2.1 Far field region/Radiation zone/Fraunhofer region

At a distance where $r >> \lambda/2\pi$, no radial component is present in the radiation field.

Thus, the radiation of electric and magnetic (far) fields, have only two field components, given by

$$E_\theta = j\frac{\beta I_m l \sin(\theta) e^{j(\omega t - \beta r)}}{4\pi\varepsilon c r} \qquad (13)$$

$$H_\phi = j\frac{\beta I_m l \sin(\theta) e^{j(\omega t - \beta r)}}{4\pi r} \qquad (14)$$

The electric field and the magnetic field lie on the spherical surface and for a small area would appear as a plane wave-traveling in the outward direction.

The ratio of E_θ *and* H_ϕ represents the intrinsic impedance, at point $P(r, \theta, \phi)$, and it is given by

$$\frac{E_\theta}{H_\phi} = \sqrt{\frac{\mu_o}{\varepsilon_o}} = 120\pi \qquad (15)$$

2.4.2.2 Near field region or Fresnel region

At a distance where r << λ/2π, the near field expression is given by equations (11) and (12). The electric dipole has to component E_r and E_θ which are both in time phase quadrature with magnetic field.

For E_θ and H_ϕ components, the near field pattern are the same as the far field patterns as both are proportional to "sin(θ)". However near-field pattern for E_r is proportional to "cos (θ)".

2.4.2.3 Reactive Near field region

That portion of near field region immediately surrounding the antenna wherein the reactive field predominates.

Quasi-stationary case (At *very low frequencies*): At low frequencies ω approaches zero. Therefore E_r, E_θ, and H_ϕ will become

$$E_r = \frac{Q_m l \cos(\theta)}{2\pi\varepsilon r^3} \qquad H_\phi = \frac{I_m l \sin(\theta)}{4\pi r^2} \qquad (16)$$

$$E_\theta = \frac{Q_m l \sin(\theta)}{4\pi\varepsilon r^3}$$

The electric fields given by equation (16) are identical to electrostatic fields of two points charges $\pm Q$ separated by a distance *l.*

The relation for magnetic field may be recognized as *Biot-Savart law* for the magnetic field of a short element current.

2.5 ANTENNA TERMINOLOGY

2.5.1 Isotropic Radiator

An isotropic radiator is an antenna which radiates uniformly in all directions. It is also called *isotropic source* or *omni directional antenna*. An isotropic antenna is a hypothetical loss less radiator, which with the practical antennas are compared. Thus, an isotropic radiator is used as reference.

Imagine that an isotropic radiator is situated at the center of a sphere of radius r. Then all energy (power) radiated from it, must be over the surface area of the sphere. Thus, the total power radiated by the source is given by

$$P_t = P_r 4\pi r^2 [Watts] \qquad (17)$$

where P_t : total radiated power, in Watts; P_r : radial power density, in Watts/m^2; r: radius of the sphere in meters.

The power density P_r on the wave front in Watts per square meter is

$$P_r = \frac{P_t}{4\pi r^2} [Watts / m^2] \qquad (18)$$

2.5.2 Radiation Pattern

The radiation pattern of an antenna is a graph which shows the variation of field strength of electromagnetic radiation at all points which are at equal distance from the antenna. It determines the distribution of radiated energy in space. The radiation pattern of an antenna is usually compared to that of theoretical source, a point-source isotropic radiator.

- If the radiation from the antenna is expressed in terms of the field strength E, the pattern is called the "*Field Strength Pattern*".
- If the radiation in a given direction is expressed in terms of power per unit solid angle, then the resulting pattern is called "*Power Pattern".*

The radiation pattern plot should be a three-dimensional plot. However, it can also be shown in two two-dimensional plots, namely the magnitude of the normalized field strength (with respect to the peak value) versus θ for a constant ϕ (the E-plane pattern); and the magnitude of the normalized field strength versus ϕ for $\theta = \pi / 2$ (the H-plane pattern). The coordinates system used is the spherical coordinates "(r, θ, ϕ) ". The antenna is assumed to be located at origin of spherical coordinates system.

The Poynting vector[1] is given by $P_r = \frac{1}{2}\mathrm{Re}(\vec{E} \times \vec{H}^*)$ (19)

Using (13) and (14), we obtain $P_r = \frac{\eta}{8}\left|\frac{I_m l}{\lambda}\right|^2 \left|\frac{\sin(\theta)^2}{r^2}\right|$ (20)

where P_r is the power density in a_r direction.

The Radiated power is proportional to sin(θ) and it is maximum when θ = 90° and minimum when θ = 0° (in the direction of axis of dipole).

[1] Poynting vector is also known as power density or power per unit area.

Now let us consider the radiation pattern of the electric field in a_θ direction from a short current element. The magnitude of "radiation term" for such antenna is given by

$$E_\theta = \frac{60\pi \, I_m l}{r\lambda} \sin\theta \qquad (21)$$

E_θ is proportional to sin(θ) .It is maximum when $\theta = 90°$ and minimum when $\theta = 0°$ (in the direction of axis of dipole).

Three dimensional pattern of a short vertical dipole is doughnut shaped (Figure 2). Figure 3 illustrates the two dimensional pattern obtained by cutting three dimensional pattern with a vertical plane. Figure 4 illustrates the two dimensional pattern obtained by cutting three dimensional pattern with a horizontal plane at the centre of the dipole.

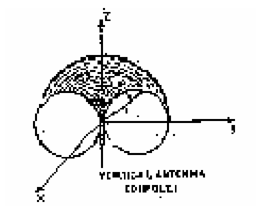


Figure 2

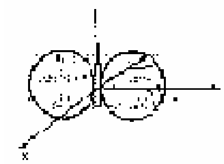


Figure 3

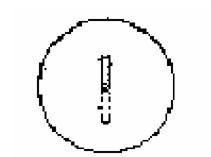


Figure 4

For a horizontal placed antenna, the patterns are shown in Figure 5, 6 and 7.

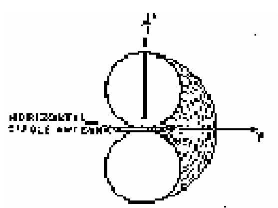


Figure 5

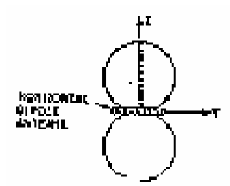


Figure 6

Figure 7

To completely specify the radiation pattern with respect to field intensity and polarization three patterns are required. They are:

- The θ component of electrical field, E_θ as a function of θ and ϕ;
- The ϕ component of electrical field, E_ϕ as a function of θ and ϕ;
- The phases of these fields as a function of the angles θ and ϕ.

2.5.3 Radiation Pattern Lobes

The performance of the antenna is usually described in terms of its principal E-plane and H-plane patterns. For linearly polarized antennas, the E-plane pattern is defined as "*the plane containing the electric field vector and the direction of maximum radiation*", and the H-plane pattern is defined as "*the plane containing the magnetic field vector and the direction of maximum radiation*".

It is practice to orient most antennas in such a way that at least one of the principal plane patterns coincide with one of the geometric planes (Figure 8). Different parts of radiation pattern are referred to as lobes. This may be sub-classified as major lobe, minor lobe, side lobe and back lobe (Figure 9):

- Major lobe: It is also called main beam and is defined as the radiation lobe containing the direction of maximum radiation.
- Minor lobe: It is any lobe except a major lobe.
- Side lobe: A side lobe is adjacent to the main lobe and occupies the hemispheres in direction of the main lobe.
- Back lobe: Normally refers to a minor lobe that occupies the hemispheres in a direction opposite to that of the major (main lobe).

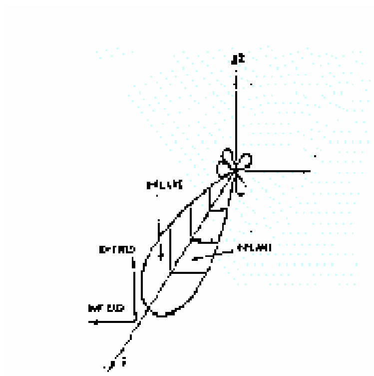


Figure 9

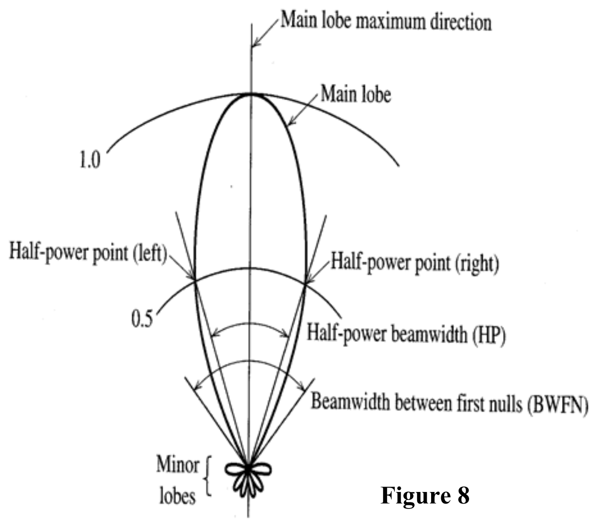


Figure 8

2.5.4 Radiation Intensity

Radiation intensity is defined as power per unit solid angle. It is a quantity that does not depend on the distance from the radiator. It is denoted by letter *U*. Unit of radiation intensity is Watts/steradian.

The radiation intensity, *U* is equal to r^2 times the magnitude of the *time average Poynting vector*.

$$U = r^2 P_{ave} \tag{22}$$

The radiation intensity can be also expressed as

$$U = U_{\max} \left| F(\theta,\phi) \right|^2 \tag{23}$$

where $U_{\max}$ and $\left| F(\theta,\phi) \right|^2$ are respectively the maximum radiation and the power pattern normalized to a maximum value of unity.

Thus, total power radiated is

$$\begin{aligned} P_t &= \iint P_{ave}.ds = \iint U(\theta,\phi) d\Omega \\ &= \iint U_{max} \left| F(\theta,\phi) \right|^2 d\Omega \end{aligned} \tag{24}$$

where dΩ = element of solid angle = $\sin\theta d\theta d\phi$.

For an isotropic radiator with uniform radiation in all directions, total solid contains 4π steradians.

$$U_{ave} = U(\theta,\phi) = \frac{P_t}{4\pi} \tag{25}$$

2.5.5 Polar Diagram

The antenna pattern is plotted on a decibel scale in polar coordinates, with intensity as the radial variable. This format permits a convenient visual interpretation of the directional distribution of the radiation lobes. This is called *polar diagram*.

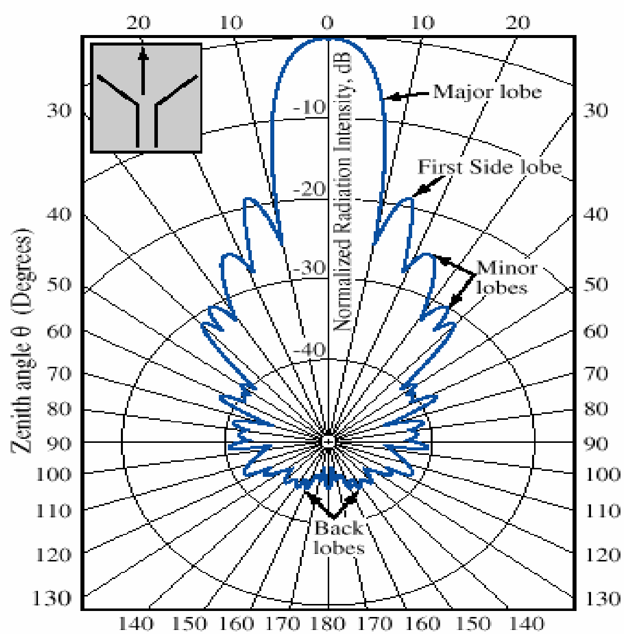


Figure 10 Polar Diagram

2.5.6 Gain

The gain of an antenna is defined as the ratio of maximum radiation intensity in given direction to the maximum radiation intensity from a reference antenna (Isotropic antenna) produced with the same power input.

$$Gain = G = \frac{U_{\max}}{U_{ave}} \tag{26}$$

where

U_{max}: Maximum Radiation intensity from test antenna;

U_{ave}: Radiation Intensity from Isotropic antenna.

Since gain denotes concentration of energy, the high values of gain are associated with narrow beam width. The dipole has a minimal transmission or reception off the ends of the antenna, but the maximum radiation exceeds that of the isotropic radiator. Since the gain comparison for antennas are made with the isotropic radiator pattern as a reference, the gain in decibel is written as dBi.

2.5.6.1 Directive Gain

The directive gain, G_d, of an antenna is defined as, in a particular, as the ratio of the power density (Poynting vector) in that particular direction at a *given distance*, to the power density that would be radiated at the *same distance* by an isotropic antenna, radiating the same total power.

$$G_d = \frac{\text{Radiation intensity in a particular direction by test antenna}}{\text{U}_{\text{ave}} : \text{Radiotion intensity of an isotropic antenna}} \quad (27)$$

2.5.6.2 Power Gain

The gain power compares the radiated power density of the actual antenna and that of an isotropic antenna on the basis of the *same input power* to both.

$$G_p = \frac{\text{Power density radiated in a particular direction by test antenna}}{\text{Power density radiated by an isotropic antenna}} \quad (28)$$

The directive gain and power gain are identical except that power gain takes into account the antenna losses.

Thus, $$G_p = \eta G_d \quad (29)$$

where η is the efficiency factor.

2.5.7 Antenna Efficiency

The efficiency of an antenna is defined as the ratio of the radiated power to the total input power supplied to the antenna and is given by

$$\eta = \frac{Radiated\ power}{Total\ input\ power} = \frac{G_p}{G_d} = \frac{P_t}{P_t + P_l} \quad (30)$$

$$\eta\% = \frac{R_r}{R_r + R_l} \times 100 \quad (31)$$

where R_r is the radiation resistance and R_l is the Ohmic loss resistance of the antenna conductor.

2.5.8 Directivity

The directivity of an antenna is a measure of its ability to direct energy in one direction in preference to radiation in other directions. It is defined as the ratio of the radiation intensity of the test antenna in a certain direction to its average radiation intensity, or

$$D(\theta,\phi) = \frac{U(\theta,\phi)}{U_{ave}} \quad (32)$$

$$D(\theta,\phi) = \frac{U(\theta,\phi)}{\frac{U_{\max}}{4\pi}\iint\left|F(\theta,\phi)\right|^2 d\Omega} \quad (33)$$

$$D(\theta,\phi) = \frac{4\pi\left|F(\theta,\phi)\right|^2}{\Omega_A} \quad (34)$$

where Ω_A is the beam solid angle[2].

This result shows that directivity is entirely determined by pattern shape.

2.5.9 Beam Solid Angle

Beam solid angle is the solid angle through which that all the power would be radiated if the power per unit solid angle (radiation intensity) equals the maximum value over the beam area Ω_A.

[2] Beam solid angle also known as beam area.

$$\Omega_A = \iint \left|F(\theta,\phi)\right|^2 d\Omega \qquad (35)$$

$$P_t = \iint U(\theta,\phi) d\Omega \qquad (36)$$

$$U = U_{max}\left|F(\theta,\phi)\right|^2 \qquad (37)$$

From (36) and (35), we see that the total radiated power can be written as

$$P_t = U_{max}\Omega_A \qquad (38)$$

Maximum directivity follows from (32) as

$$D_{\max} = \frac{U_{\max}}{U_{ave}} = \frac{U_{\max}}{P/4\pi} = \frac{4\pi}{\Omega_A} \qquad (39)$$

Also from (32) and (23), we see that

$$D(\theta,\phi) = \frac{U_{\max}\left|F(\theta,\phi)\right|^2}{U_{ave}} = D_{\max}\left|F(\theta,\phi)\right|^2 \qquad (40)$$

Since $\left|F(\theta,\phi)\right|^2$ can have a maximum value of unity, the maximum value of directivity is

$$D_{max} = \frac{U_{max}}{U_{ave}}$$

2.5.10 Half Power Beamwidth (HPBW)

It is the angular separation between the points on power pattern where the power value is one-half the maximum value. In case of field intensity pattern the angular separation between 0.707 times the maximum intensity points will be the HPBW.

The half power beam width values can be used to find *approximate directivity* of an antenna neglecting the minor lobes.

$$\mathrm{D}_{\mathrm{approx}} = \frac{4\pi}{\theta_{\mathrm{HP}}\phi_{\mathrm{HP}}} = \frac{41000}{\theta^{\mathrm{o}}_{\mathrm{HP}}\phi^{\mathrm{o}}_{\mathrm{Hp}}} \qquad (41)$$

where $\theta_{\mathrm{HP}}, \phi_{\mathrm{HP}}$ are half power beam widths in radians and $\theta^{\mathrm{o}}{}_{\mathrm{HP}}, \phi^{\mathrm{o}}{}_{\mathrm{HP}}$ are in degrees.

2.5.11 Side Lobe Level

Side lobes refer to the regions of unwanted radiation that are normally found surrounding the main beam. Side lobe level is usually referred to the level of the highest side lobe, which is normally the nearest side lobe to the main beam.

In modern radar application, it is required that the side lobe levels are of the order of minus 40 or more decibels from the level of main beam.

2.5.12 Radiation Resistance

R_r of an antenna is the hypothetical resistance that would dissipate the same amount of power as the radiated power P_r. We can find R_r from $P_r = \frac{1}{2} I^2 R_r$ where the current I in the resistance is equal to the maximum current flowing along the antenna.

The radiation resistance of short dipole $= 80\pi^2\left(\frac{l}{\lambda}\right)^2$ ohms (42)

The radiation resistance of $\frac{\lambda}{2}$ antenna[3] = 73 ohms (43)

2.5.13 Effective Area/Effective Aperture/Capture Area

In the transmitting mode of antennas, the time varying current and charges radiate electromagnetic waves, which carry energy, from a transmitting antenna to a receiving antenna.

The receiving antenna extracts energy from an incident electromagnetic wave and delivers it to a load. The concept of *effective aperture* is best understood by considering an antenna to have an area which extracts electromagnetic energy from an incident electromagnetic wave. It may be defined as the ratio of power received at the antenna load terminal to the Poynting vector (or power density) in Watts/m^2 of the incident wave.

Thus, $$A_e = \frac{P}{P_{ave}} \qquad (44)$$

where

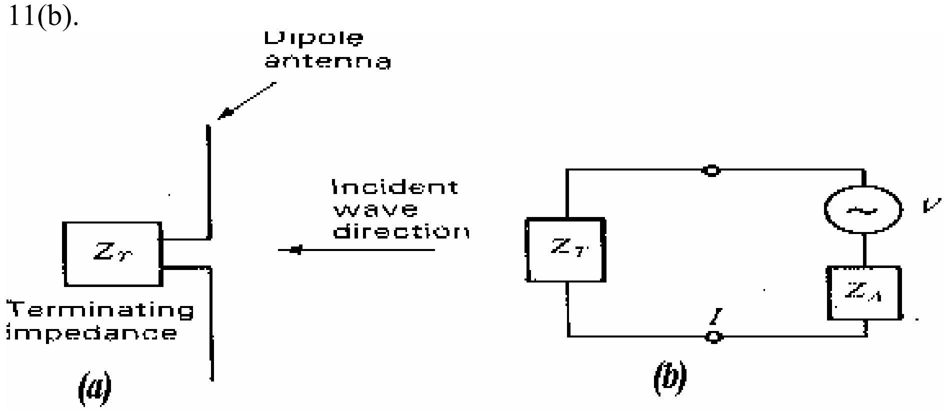
P = Power received in Watts;

P_{ave} = average Poynting vector of incident wave in Watts/m^2 .A_e = Effective area in m^2 .

Let a receiving antenna be placed in the field of a plane polarized wave as in Figure 11(a).

The receiving antenna (dipole) is terminated in load impedance, $Z_L = R_L + jX_L$.

Since antenna extracts energy from incident electromagnetic waves, delivers the same to terminated load impedance, Z_L, this entire system can be replaced by an equivalent circuit given in Figure 11(b).



Schematic diagram of dipole antenna terminated in impedance Z_T with plane wave incident on antenna (*a*) and equivalent circuit (*b*).

Figure 11

V = Equivalent open circuit or Thevenin voltage.

$Z_A = R_A + jX_A$ = Equivalent Thevenin's impedance and $R_A = R_r + R_\ell$ (radiation resistance + loss resistance).

[3] This radiaton resistance can not be found simply from equation (42) since it is not a short dipole.

The induced current through the terminal load is $I_{rms} = \frac{V}{Z_L + Z_A}$.

If, $R_\ell = 0$, i.e., the antenna is lossless,

then $$I_{rms} = \frac{V}{R_L + jX_L + R_A + jX_A} \tag{45}$$

The power received by the antenna is given by

$$P = \frac{V^2 R_L}{(R_L + R_A)^2 + (X_L + X_A)^2}$$

Hence the effective area A_e can be written as

$$A_e = \frac{V^2 R_L}{\left[(R_L + R_A)^2 + (X_L + X_A)^2\right] P_{ave}} \tag{46}$$

If the load impedance is matched to the internal impedance,

then $$Z_L = Z_A^* = R_A - jX_A$$

and the maximum effective area will be

$$(A_e)_{\max} = \frac{V^2}{4 R_r P_{ave}} \tag{47}$$

2.5.14 Physical Aperture

The physical aperture is related to the actual physical size (or cross-section) of the antenna and is denoted by A_p. The physical aperture may be defined as "the cross-section perpendicular to the direction of propagation of incident electromagnetic wave with antenna set for maximum response".

If no losses are there, $A_p = A_e$; $D = \frac{4\pi}{\Omega_A}$ and $\lambda^2 = A_e \Omega_A$ where Ω_A is the beam area.

From these, we get

$$D = \frac{4\pi}{\lambda^2} A_e \tag{48}$$

This is the directivity and the effective aperture relation.

2.5.15 Effective Length

The term effective length of an antenna represents the effectiveness of an antenna as radiator or as collector of electromagnetic wave energy. For a receiving antenna, the effective length is the ratio of the induced voltage at the terminal of the receiving antenna under open-circuit condition to the incident electric field intensity E.

$$l_e = \frac{V}{E} \text{ (meters)} \tag{49}$$

In terms of the maximum effective aperture, the effective length can be written as

$$l_e = 2\sqrt{\frac{R_r (A_e)_{\max}}{377}} \text{ (meters)} \tag{50}$$

Now for transmitting antenna, the effective length is that of an equivalent linear antenna that has the same current I (as the terminal of the actual antenna) at all the point along its length and that radiates the same field intensity E as the actual antenna.
I = current at the terminals of the actual antenna; I(z) = current at any point z of the antenna;
l_e = effective length; and l = physical length.

Hence, for transmitting antenna, the effective length is given by

$$\ell_e = \frac{1}{I} \int_{-\ell/2}^{\ell/2} I(z) dz$$

2.6 RECIPROCITY THEOREM

The reciprocity theorem for antenna is stated as follows
"If an e.m.f. is applied to the terminals of an antenna no.1 and the current measured at the terminals of antenna no.2, then an equal current both in amplitude and phase will be obtained at the terminals of antenna no.1, if the same e.m.f. is applied to the terminals of antennas no.2."

OR

"If a current I_1, at the terminals of antenna no.1 induces an e.m.f. E_{21} at the open terminals of antenna no.2 and a current I_2 at the terminals of antenna no.2 induces an e.m.f. E_{12} at the open terminals of antenna no.1, then $E_{12} = E_{21}$, provided $I_1 = I_2$".

A transmitter of frequency *f* and zero impedance is connected to the terminals of antenna no.2, which is generating a current I_2 and inducing an e.m.f. E_{12} at the open terminals of antenna no.1. Now, the same transmitter is transferred to antenna no. 1 which is generating a current I_1 and inducing an e.m.f. E_{21} at the open terminals of antenna no.2.

According to the statement of *reciprocity theorem*, **$I_1 = I_2$ provided $E_{12} = E_{21}$.** The ratio of an e.m.f. to the current is an impedance, therefore, the ratio E_{12} / I_2 is given the name: *transfer impedance Z_{12}*, and so also E_{21} / I_1 as *transfer impedance Z_{21}*. From reciprocity, it follows that the two transfer impedances are equal: $Z_{12} = Z_{21}$. This is called the *mutual impedance*, Z_m, between the two antennas. Therefore, $Z_m = Z_{12} = Z_{21}$.

2.7 FRIIS'S TRANSMISSION FORMULA

The two antennas shown in Figure 12 are part of a free-space communication link, with the separation between the antennas, *R*, being large enough for each antenna to be in the free-field region of the other. The transmitting and receiving antennas have effective areas A_t, and A_r and radiation efficiencies ξ_t and ξ_r, respectively.

Our objective is to find a relationship between P_t, the transmitter power supplied to the transmitting antenna, and P_{rec}, the power delivered to the receiver by the receiving antenna. As always, we assume that both antennas are impedance matched to their respective transmission lines.

Initially, we shall consider the case where the two antennas are oriented such that the peak of the radiation pattern of each antenna points in the direction of the other. The average power density at receiver (for maximum reception) is

$$S = \frac{P_t}{4\pi R^2} \qquad (51)$$

Power intercepted by the receiving antenna with an effective area A_r is

$$P_{int} = S A_r \tag{52}$$

The transmitting antenna has effective aperture A_t. So, directivity is $D = 4\pi A_t/\lambda^2$.
Hence, power available at receiver is

$$P_r = DS A_r$$

$$= S A_r 4\pi A_t/\lambda^2 = \frac{P_t A_r}{4\pi r^2}\frac{4\pi A_t}{\lambda^2} = \frac{P_t A_r A_t}{r^2\lambda^2} \tag{53}$$

$$\frac{P_r}{P_t} = \frac{A_r A_t}{r^2\lambda^2} \tag{54}$$

The relation is known as the *Friis's Transmission Formula*, and P_{rec}/P_t is sometimes called the *power transfer ratio*.

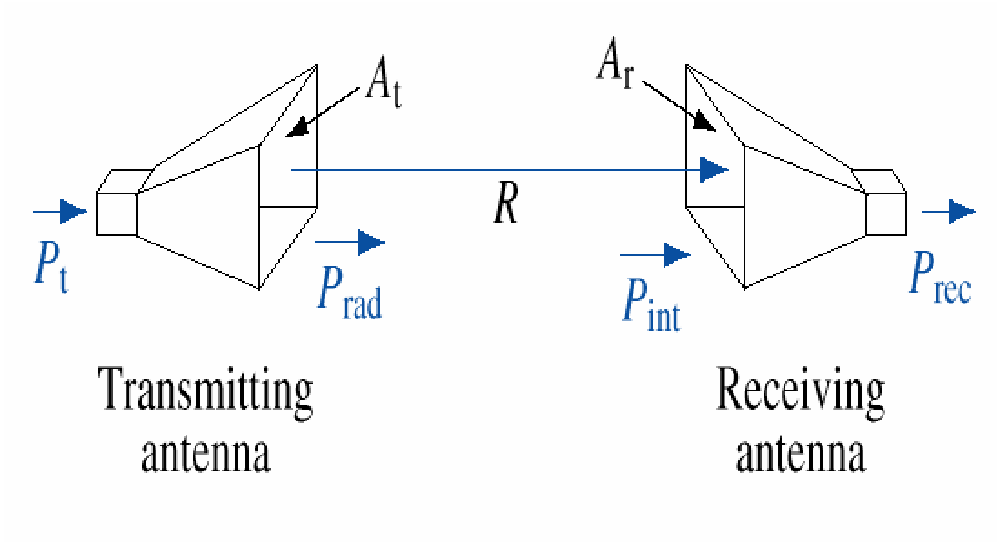
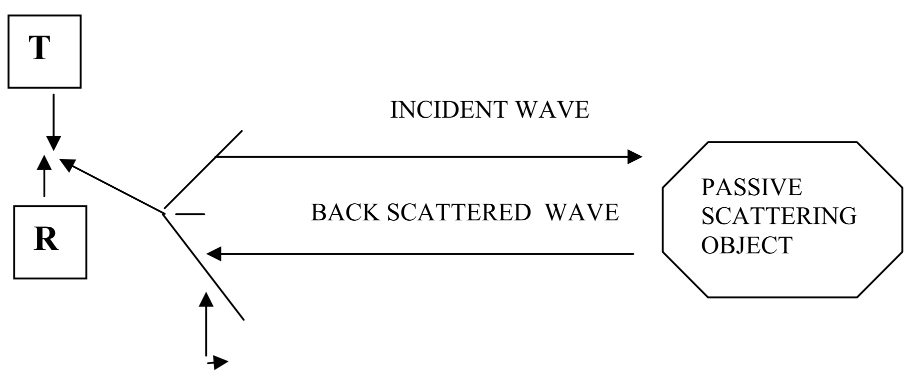


Figure 12

2.8 RADAR EQUATION

Basic arrangement shown in Figure 13 below.



Transmitting - Receiving Antenna

Figure 13

A transmitter connected to transmitting antenna will send a pulse. The pulse strikes a passive object .The power intercepted by the object is given by Friis's formula,

$P_{int(object)} = \dfrac{P_t A_t}{r^2 \lambda^2} \sigma$ where σ =radar cross-section of object m^2.

Assuming that the object scatters energy in all directions uniformly, that is D=1, its effective aperture will be $\dfrac{\lambda^2}{4\pi}$. The scattered power received back at the transmitter location is given by use of Friis's formula once again as

$$P_{r(\text{by antenna})} = \frac{P_{int(object)} A_t}{r^2 \lambda^2} \frac{\lambda^2}{4\pi}$$

Substituting for $P_{int(object)}$ and rearranging, we yield

$$\frac{P_{r(antenna)}}{P_t} = \frac{A^2 \sigma}{4\pi r^4 \lambda^2}$$

This is the *radar equation*.

2.9 RADIATION PATTERN OF A THIN WIRE ANTENNA

In the case of short dipole, we have assumed constant current distribution on the short electric dipole. But it has been found numerically and confirmed experimentally that the current on thin wire antennas is approximately sinusoidal.

We will use such approximation to analyze the radiation fields of a linear thin wire antenna. Thus, consider the linear wire antenna shown in Figure 14.

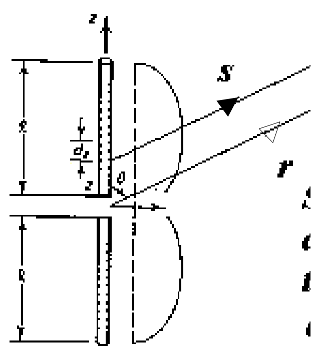


Figure 14

The sinusoidal current distribution may be assumed as

$$I(z) = I_m \sin \beta(l - |z|) \quad -l < z < l \qquad (54)$$

From this current, we can calculate the radiation pattern since it is a z-directed line source, by regarding the antenna as made up of a series of *infinitesimal dipoles* of length *dz*. The field of the entire antenna may then be obtained by integrating the fields from all of the dipoles making up the antenna.

The far fields dE_θ and dH_ϕ at a distance s from the *infinitesimal dipole dz* are integrated to get the total fields as

$$E_\theta = \int_{-l}^{l} j\frac{60\pi \quad I_{retarded} \sin(\theta)dz}{s\lambda} \tag{55}$$

$$H_\phi = \int_{-l}^{l} j\frac{I_{retarded}\sin(\theta)dz}{2s\lambda} \tag{56}$$

Introducing the value of $I_{retarded}$ from (54) in (56), we have[4]

$$H_\phi = \frac{jI_m \sin\theta e^{j\omega t}}{2\lambda}\left[\int_{-l}^{0}\frac{1}{s}\sin(\beta(l+z))e^{-j\beta s}dz + \int_{0}^{l}\frac{1}{s}\sin(\beta(l-z))e^{-j\beta s}dz\right] \tag{57}$$

At large distance the effect between s and r can be neglected as in its effect on the amplitude although its effect on the phase must be considered.

From Figure 14, $s = r - z\cos\theta$ (58)

Substituting (58) in (57) and also s=r for the amplitude factor, (57) becomes

$$H_\phi = \frac{jI_m \sin\theta e^{j(\omega t-\beta r)}}{2\lambda r}\left[\int_{-l}^{0}\sin(\beta(l+z))e^{j(\beta\cos\theta)z}dz + \int_{0}^{l}\sin(\beta(l-z))e^{j\beta(\cos\theta)z}dz\right] \tag{59}$$

Thus the magnetic field in ϕ-direction follows as

$$H_\phi = \frac{jI_{retarded}}{2\pi r}\left[\frac{\cos(\beta l\cos\theta)-\cos(\beta l)}{\sin\theta}\right] \tag{60}$$

Multiplying H_φ by 120π gives

$$E_\theta = \frac{j60I_{retarded}}{r}\left[\frac{\cos(\beta l\cos\theta)-\cos(\beta l)}{\sin\theta}\right] \tag{61}$$

The time average power density may be obtained from equation (60) and (61).

Thus,
$$P_r = \frac{15|I_m|^2}{\pi r^2}\left[\frac{\cos(\beta l\cos\theta)-\cos(\beta l)}{\sin\theta}\right]^2 \tag{62}$$

[4] Equation (57) assumes the total length of linear wire antenna is 2l.

and $$F_{\theta} = \left[\frac{\cos[(\beta l \cos\theta) - \cos(\beta l)]}{\sin\theta}\right] \qquad (63)$$

The factor F(θ) represents the relative pattern radiation as a function of θ.

For the specific case of half wave length antenna, $2l = \frac{\lambda}{2}$ and $\beta l = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \frac{\pi}{2}$

The factor F(θ) will become $F(\theta) = \frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}$ (64)

2.10 ANTENNA ARRAYS

To produce desired directional radiation pattern, several antennas can be arranged in space and interconnected. Such a configuration of multiple radiating elements is referred to as an *array antenna*, or *array*.

Many small antennas can be used in an array to obtain a level of performance similar to that of a single large antenna. Arrays are found in many geometrical configurations. The most elementary is that of a linear array in which the array element centres lie along a straight line.

Case 1: Two Isotropic Point Sources with Identical Amplitude and Phase Currents, and Spaced One-Half Wavelength Apart

If we use phases corresponding to the path length differences shown in Figure 15, the array factor is

$$AF = Ee^{-j\psi/2} + Ee^{+j\psi/2} = 2E\cos(\psi/2)$$

where $\psi = \beta d\cos\theta$.
When $d = \lambda/2$ and $\beta = 2\pi/\lambda$, $\beta d = \pi$

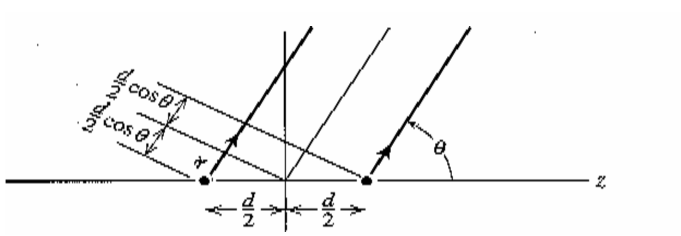


Figure 15

The array factor *AF* becomes

$$AF = 2E\cos(\tfrac{\pi}{2}\cos\theta)$$

Normalizing the array factor for a maximum value of unity gives

$$f(\theta) = \cos(\tfrac{\pi}{2}\cos\theta) \qquad (65)$$

The polar plot of the array is given in Figure 16.

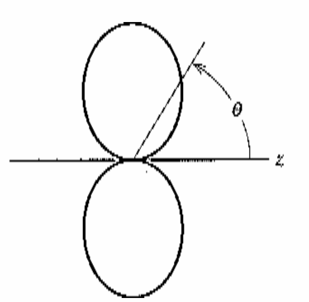


Figure 16

Case 2: Two Isotropic Point Sources with Identical Amplitudes and Opposite Phases, and Spaced One-Half Wavelength Apart

The arrangement is still same as Case 1, as shown in Figure 15. The left source is 180° out-of-phase with respect to the right source. Taking the centre of sources or origin as reference for phase, we can take radiation from right source is leading by 90^0 and from left source is lagging by 90^0.

If we use phases corresponding to the path length differences shown in Figure 15, the array factor is

$$AF = -Ee^{-j\psi/2} + Ee^{+j\psi/2} = 2jE\sin(\psi/2)$$

Using $\frac{\beta d}{2} = \frac{\pi}{2}$, the array factor *AF* becomes

$$AF = 2jE\sin(\tfrac{\pi}{2}\cos\theta)$$

Normalizing the array factor for a maximum value of unity gives

$$f(\theta) = \sin(\tfrac{\pi}{2}\cos\theta) \quad (66)$$

From this, the pattern can be sketched, yielding a plot as shown in Figure 17.

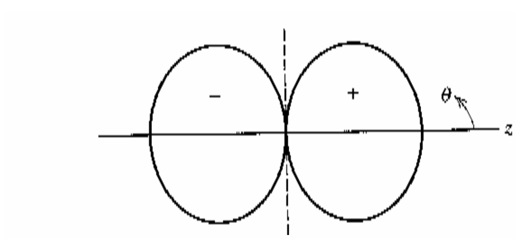


Figure 17

2.11 PATTERN MULTIPLICATION

The *total field pattern* of an array of non-isotropic but similar sources is the product of the individual source pattern and the pattern of an array of isotropic point sources, each located at the phase centre of individual source with relative amplitude and phase of the source; while the *total phase pattern* is the sum of the phase patterns of the individual sources and array of isotropic point sources. This is called *the principle of pattern multiplication*.

E (total) = E (source pattern) × E (isotropic array pattern)

Example 1:
Consider **two collinear short dipoles spaced a half-wavelength apart and equally excited.** The element pattern is sinθ for an element along the z-axis and the array factor was found to be $\cos(\frac{\psi}{2})$.

Here, Total pattern = sinθ X $\cos(\frac{\psi}{2})$

where $\psi = \beta d \cos\theta = \pi \cos\theta$

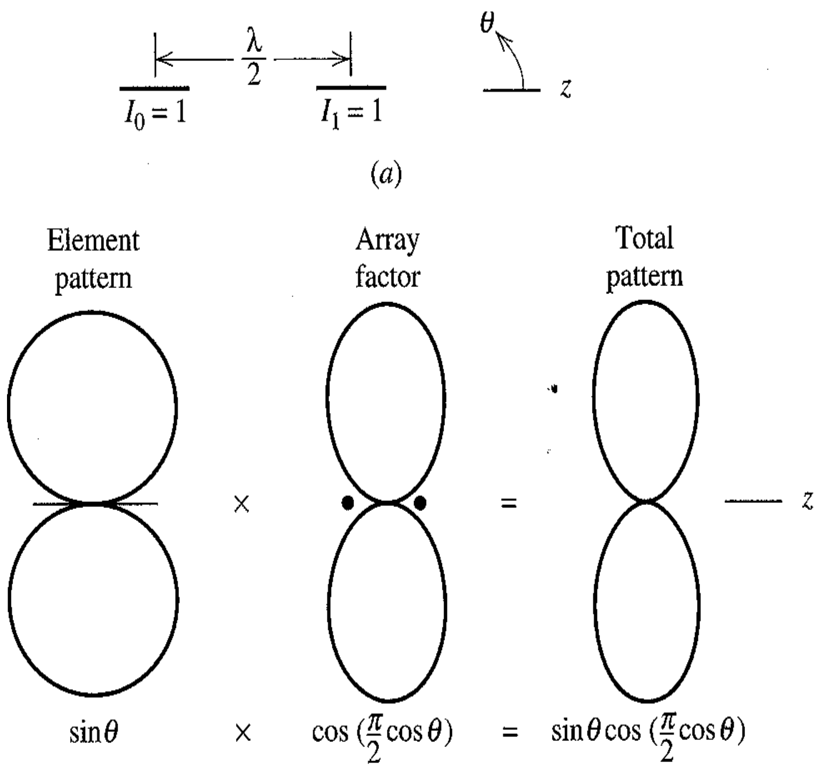


Figure 18 Array of two half-wavelength spaced, equal amplitude, equal phase, and collinear short dipoles

Example 2:

Consider **two Parallel, Half-Wavelength Spaced Short Dipoles**
The complete pattern for the array of two parallel short dipoles in is found by pattern multiplication as indicated.

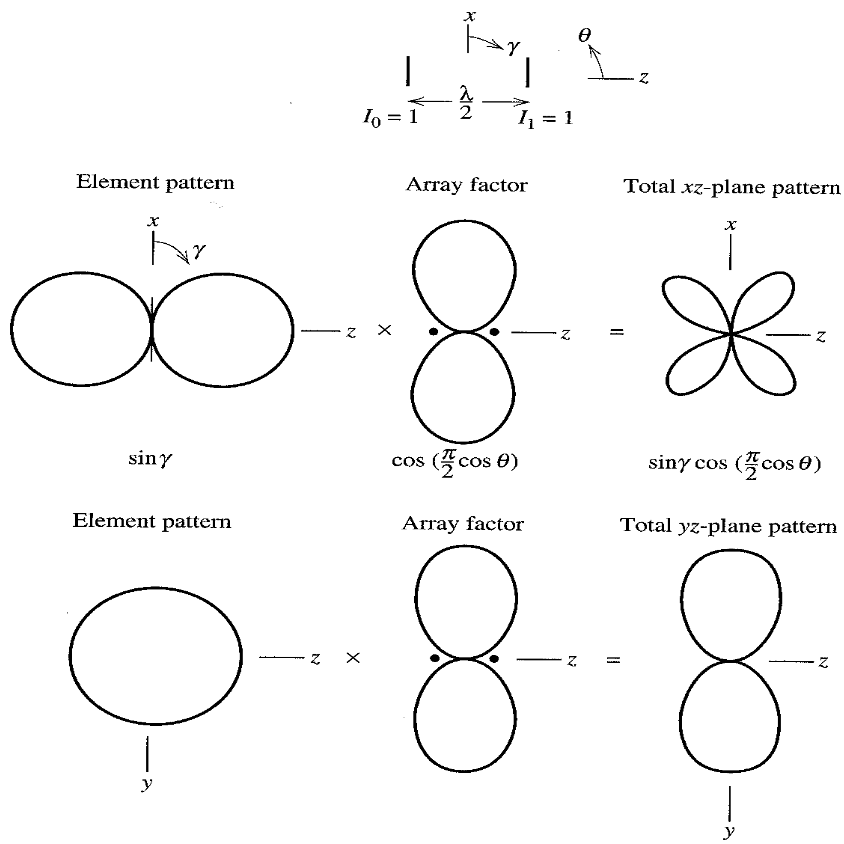


Figure 19 Array of two half-wavelength spaced, equal amplitude, equal phase parallelshort dipoles. From top to bottom: (a) The array; (b) The xz-plane pattern; (c) The yz-plane pattern.

2.12 LINEAR ARRAY OF N ISOTROPIC POINT SOURCES OF EQUAL AMPLITUDE AND SPACING

Consider a linear array of n isotropic point sources of equal amplitude and spacing. At a large distance in direction θ, the total field is given by

$$E = 1 + e^{j\psi} + e^{j2\psi} + ... + e^{j(n-1)\psi} \qquad (67)$$

where ψ is the phase difference of the field radiated in the θ direction from adjacent sources and is given by

$$\psi = \frac{2\pi d}{\lambda}\cos\theta + \delta \qquad (68)$$

where d= spacing between sources and δ is phase difference between adjacent sources. In this case, source 1 is taken as reference for phase and amplitude of each source is taken as 1.
Multiply (67) by $e^{j\psi}$, we yield

$$Ee^{j\psi} = e^{j\psi} + e^{j2\psi} + ... + e^{j(n-1)\psi} + e^{jn\psi} \qquad (69)$$

Subtract (69) from (67), we yield

$$E(1-e^{j\psi}) = 1-e^{jn\psi}$$

and $$E = \frac{1 - e^{jn\psi}}{1 - e^{j\psi}} \tag{70}$$

Rearranging it, we yield

$$E = \frac{e^{jn\psi/2}}{e^{j\psi/2}}\left(\frac{e^{jn\psi/2} - e^{-jn\psi/2}}{e^{j\psi/2} - e^{-j\psi/2}}\right) = e^{j\xi}\frac{\sin(n\psi/2)}{\sin(\psi/2)}$$

$$= \frac{\sin(n\psi/2)}{\sin(\psi/2)}\angle\xi \tag{71}$$

where $\xi = \frac{n-1}{2}\psi$ is the phase angle of E referred to field from source 1.

If centre point of the array is taken as reference for phase, then $\xi=0$ and

$$\text{Array pattern} = \frac{\sin(n\psi/2)}{\sin(\psi/2)}.$$

For $\psi=0$, we get E=n. This is the maximum value that E can attain. $\therefore$ $E_{max} = n$.

Therefore, normalised value of total field,

$$E_n = \frac{1}{n}\frac{\sin(n\psi/2)}{\sin(\psi/2)} \tag{72}$$

In this case the maximum radiation occurs in a direction θ_m such that $\beta d \cos\theta_m + \xi = 0$.

When maximum radiation occurs in a direction perpendicular to the line of array, i.e. in the direction $\theta = \pm\pi/2$, it is called *broadside array*. This happens when $\xi=0$. If the maximum radiation occurs in the direction $\theta=0$, it is called *end-fire array*. For this, $\xi= -\beta d$.