Assignment 1: Answer

Week Assigned: Week 4
Due date: Week 5 before 12:30 p.m. Friday, 18 January 2002

1. (a) Entropy of the source, H(S) = 1.485 bits/symbol
   (b) Entropy of the second-order extension of the source H(S^2) = 2.971 bits/symbol

2. First Case: Moving a "combined symbol" as *high* as possible

For source S, the code words are:
S_0=1; S_1=00; S_2=01

Average code word length, $\widetilde{L}$ = 1.5 bits/symbol
Entropy of the source, H(S) = 1.486 bits/symbol
Efficiency, η = 99.07%

For second-order extension of the source S^2,
For source S, the code words are:
S_0S_0=10; S_0S_1=001; S_1S_0=010; S_0S_2=110; S_2S_0=111; S_1S_1=0000; S_1S_2=0001; S_2S_1=0110; S_2S_2=0111;

Average code word length, $\widetilde{L}$ = 3 bits/symbol
Entropy of the source, H(S^2) = 2.971 bits/symbol
Efficiency, η = 99.03%

Hence, shown that coding efficiency for S (99.07%) is equal to coding efficiency for S^2 (99.03%).

Second Case: Moving a "combined symbol" as *low* as possible
For source S, the code words are:
S_0=0; S_1=10; S_2=11

Average code word length, $\widetilde{L}$ = 1.5 bits/symbol
Entropy of the source, H(S) = 1.486 bits/symbol
Efficiency, η = 99.07%

For second-order extension of the source S^2,
For source S, the code words are:
S_0S_0=01; S_0S_1=000; S_1S_0=001; S_0S_2=101; S_2S_0=110; S_1S_1=1000; S_1S_2=1001; S_2S_1=1110; S_2S_2=1111;

Average code word length, $\widetilde{L}$ = 3 bits/symbol
Entropy of the source, H(S^2) = 2.971 bits/symbol
Efficiency, η = 99.03%

Hence, shown that again coding efficiency for S (99.07%) is equal to coding efficiency for S^2 (99.03%).

3. (a) Information in dot, I(dot) = 2 bits
   Information in dash, I(dash) = 0.415 bits
   (b) Average information in the dot-dash code, H = 0.81125 bit/symbol
   (c) Average information rate = 32.45 bits/second

4. The MATHLAB program is as below:

```
for n = 1:4
    a(n) = randsrc (1,1,[97,98,99,100,101,102,103,104,105,106,107,108,109,110,111,
112,113,114,115,116,117,118,119,120,121,122])
     b(n) = char(a(n))
end
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   To get a four-letter word in English dictionary, the above program need to be ran more than hundred times. For instance, we need to run 326 times to get the word "west".

   The entropy of the random 4-letter word = 0.723 bits/symbol