MULTIMEDIA UNIVERSITY
FACULTY OF ENGINEERING
PEM2046 ENGINEERING MATHEMATICS IV
TUTORIAL

A. Linear Programming (LP)

1. Identify the optimal solution and value:

(a) Maximize $f = 20x_1 + 30x_2$

st.
$$-x_1 + x_2 \geq -1$$
$$x_1 + x_2 \leq 6$$
$$x_2 \leq 5,$$
$$x_1 \geq 0 \text{ and } x_2 \geq 0$$

(b) Minimize $f = 45x_1 + 22.5x_2$

st
$$-x_1 + x_2 \geq -5$$
$$2x_1 + x_2 \geq 10$$
$$x_2 \geq 4$$
$$10x_1 + 15x_2 \leq 150$$
$$x_1 \geq 0 \text{ and } x_2 \geq 0$$

[(1,5), 170; (3–3t, 4+6t), $0 \leq t \leq 1$, 225]

2. Giapetto's Woodcarving, Inc., manufactures two types of wooden toys: Soldiers and trains. A Soldier sells for \$ 27 and uses \$ 10 worth of raw materials. Each Soldier that is manufactured increases Giapetto's variable labor and overhead by \$ 14. A train sells for \$ 21 and uses \$ 9 worth of raw materials. Each train built increase Giapetto's variable labor and overhead by \$ 10. The manufacturer of wooden soldiers and trains requires two types of skilled labor: carpentry and finishing. A soldier requires 2 hours of finishing labor and 1 hour of carpentry labor. A train requires 1 hours of finishing labor and 1 hour of carpentry labor. Each week, Giapetto can obtain all the needed raw materials but only 100 Finish hours and 80 carpentry hours. Demand for trains is unlimited, but at most 40 soldiers are bought each week. Giapetto wants to maximize weekly profit (revenues-costs). **Formulate** a mathematical model of Giapetto's situation that can be used to maximize Giapetto's weekly profit.

3. An auto company manufactures cars and trucks. Each vehicle must be processed in the paint shop and body assembly shop. If the paint shop were only painting trucks, 40 per day could be painted. If the paint shop were only painting cars, 60 cars could be painted. If the body shop were only producing cars, it could process 50 per day. If the body shop were only producing trucks, it could process 50 per day. Each truck contributes \$ 300 to profit, and each car contributes \$ 200 to profit.
   (a) Use graphical method to determine a daily production schedule that will maximize the company's profit.
   (b) Suppose that auto dealers requires that the Auto Company in the previous example produce at least 30 trucks and 20 cars. Find the optimal solution to this new LP.

[12000; Infeasible LP]

4. Refer to the feasible region in question 1(a), write down all the vertices of the feasible region. Explain how to use the graphical solution to deduce the number of different basic feasible solutions.

5. Consider the following linear program

Maximize $z = 2x_1 + 3x_2 + x_3$
Subject to $x_1 + 2x_2 + 2x_3 \leq 3$
$2x_1 + 3x_2 + 4x_3 \leq 6$
with all variables nonnegative.

(a) Given that s_1 and s_2 are slack variables for constraint 1 and 2 respectively.

Determine whether
(i) $[x_1, x_2, x_3, s_1, s_2] = [1,0,0,2,4]$
(ii) $[x_1, x_2, x_3, s_1, s_2] = [1,0,1,0,0]$
is a basic feasible solution to the linear program given above.

(b) Write down the basic feasible solution corresponding to basic variables x_1 and x_2, and the corresponding vertex of the feasible region.

(c) Produce a different basic feasible solution to the linear program given above.

6. Consider the following problem.

$$\begin{aligned} \text{Maximize } z &= 2x_1 + 2x_2 + 3x_3 \\ \text{s.t.} \quad 2x_1 + x_2 + 2x_3 &\leq 4 \quad (1) \\ x_1 + x_2 + x_3 &\leq 3 \quad (2) \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

Let x_4 and x_5 be the slack variables of constraint (1) & (2). If you are given the information that the Simplex iteration proceeds as follow to obtain the optimal solution in two iteration:

- The entering variable is x_3 and the leaving variable is x_4;
- The entering variable is x_2 and the leaving variable is x_5;

(a) By using the information provided, find the optimal solution. (DO NOT USE SIMPLEX ALGORITHME) [(0, 2, 1), 7]
(b) Develop a three-dimensional drawing of the feasible region for this problem, and show the path followed by Simplex method.

7. Solve the following linear-programming problems by means of the simplex method:

(a) Maximize $z = 2x_1 + 4x_2 + 3x_3$

st
$$3x_1 + 4x_2 + 2x_3 \le 60$$
$$2x_1 + x_2 + 2x_3 \le 40$$
$$x_1 + 3x_2 + 2x_3 \le 80$$
$$x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0$$

(b) Maximize $z = 4x_1 + 3x_2 + 6x_3$

st
$$3x_1 + x_2 + 3x_3 \le 30$$
$$2x_1 + 2x_2 + 3x_3 \le 40$$
$$x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0$$

(c) Maximize $4y_1 + 5y_2$

s.t.
$$-y_1 + y_2 \le 4$$
$$y_1 - y_2 \le 10$$
$$y_1, y_2 \ge 0$$

(d) Minimize $2y_1 - 3y_2$

s.t.
$$y_1 + y_2 \le 4$$
$$y_1 - y_2 \le 6$$
$$y_1, y_2 \ge 0$$

(e) Minimize $z = 4x_1 - 10x_2 - 20x_3$

s.t.
$$3x_1 + 4x_2 + 5x_3 \le 60$$
$$2x_1 + x_2 \le 20$$
$$2x_1 + 3x_3 \le 30$$
$$x_1, x_2, x_3 \ge 0$$

(f) Maximize $30x_1 - 4x_2$

s.t.
$$5x_1 - x_2 \le 30$$
$$x_1 \le 5$$
$x_1 \ge 0$, x_2 .urs

[(0, 6 2/3, 16 2/3), :z=76 2/3; (0,10,6*2/3), z =70; unbounded LP; (0,4), –12; (0, 2.5, 10), –225; (5, -5), 170]

8. Suppose we have obtained the tableau as follow for a maximization problem. State conditions on a_1, a_2, b, c_1, c_2 (if applicable) that required to make the following statements true:

Basic	z	x_1	x_2	x_3	x_4	x_5	x_6	Solution
z	1	c_1	c_2	0	0	2	0	10
	0	4	a_1	1	0	3	0	b
	0	–1	–5	0	1	–1	0	2
	0	a_2	–3	0	0	–4	1	3

(a) The current solution is optimal, and there are alternative optimal solutions
(b) The current basic solution is not a basic feasible solution
(c) The current basic solution is a degenerate basic feasible solution.
(d) The current basic solution is feasible, but the LP is unbounded.
(e) The current basic solution is feasible, but the objective function value can be improved by replacing x_6 as a basic variable with x_1.

9. Convert the following problem to its dual problem:

(a) Minimize: $z = 15x_1 + 12x_2$

st

$$
\begin{aligned}
x_1 + x_2 &\geq 1.5 \\
2x_1 + 3x_2 &\geq 5 \\
x_1,\ x_2 &\geq 0
\end{aligned}
$$

(b) Maximize: $z = 5x_1 + 12x_2 + 4x_3$

st

$$
\begin{aligned}
x_1 + 2x_2 + x_3 &\leq 10 \\
2x_1 - x_2 + 3x_3 &= 8 \\
x_1,\ x_2,\ x_3 &\geq 0
\end{aligned}
$$

(c) Maximize: $z = 4x_1 - x_2 + 2x_3$

st

$$
\begin{aligned}
x_1 + x_2 &\leq 5 \\
2x_1 + x_2 &\leq 7 \\
2x_2 + x_3 &\geq 6 \\
x_1 + x_3 &= 4 \\
x_1 \geq 0,\ x_2,\ x_3 &\text{ .urs}
\end{aligned}
$$

(d) Minimize: $w = 4y_1 + 2y_2 - y_3$

st

$$
\begin{aligned}
y_1 + 2y_2 &\leq 6 \\
y_1 - y_2 + 2y_3 &= 7 \\
y_1, y_2 \geq 0,\ y_3 &\text{ .urs}
\end{aligned}
$$

10. Consider the following linear programming problem:

$$
\begin{aligned}
\text{Max} \quad & z = 4x_1 + x_2 \\
\text{s.t.} \quad & 3x_1 + 2x_2 \leq 6 \\
& 6x_1 + 3x_2 \leq 10 \\
& x_1, x_2 \geq 0
\end{aligned}
$$

Suppose that in solving this problem, row 0 of the optimal tableau is found to be $z + 2x_2 + s_2 = 20/3$. Use the Dual theorem to prove that the computations must be incorrect.

11. Consider the following LP allocation model:

$$
\begin{aligned}
\text{Maximize:} \quad & 3x_1 + 2x_2 \\
\text{Subject to:} \quad & 4x_1 + 3x_2 \leq 12 \quad \text{(resource 1)} \\
& 4x_1 + x_2 \leq 8 \quad \text{(resource 2)} \\
& 4x_1 - x_2 \leq 8 \quad \text{(resource 3)} \\
& x_1,\ x_2 \geq 0
\end{aligned}
$$

with the optimum tableau given by:

Basic	z	x_1	x_2	s_1	s_2	s_3	Solution
z	1	0	0	5/8	1/8	0	17/2
x_2	0	0	1	1/2	–1/2	0	2
x_1	0	1	0	–1/8	3/8	0	3/2
s_3	0	0	0	1	–2	1	4

(a) If there is a chance to increase the availability of resources. Which resource should be given the priority for an increase in level?

(b) The range of feasibility is defined as the range over which the right hand side quantities can change without affecting the optimal solution. Find the range of feasibility of resource 1 & 3.

(c) In each of the following cases, indicate whether the given solution remains optimal. If yes, identify the new optimal value.

(i) The profit coefficients change from [3 2] to [4 3].
(ii) The profit coefficients change from [3 2] to [5 4].
(iii) The RHS of the constraints change from $[12\ \ 8\ \ 8]^T$ to $[15\ \ 10\ \ 5\ ]^T$.
(iv) The RHS of the constraints change from $[12\ \ 8\ \ 8]^T$ to $[8\ \ 10\ \ 10\ ]^T$.

[(a) 1 (b) 8 to 24, 4 to ∞ (c) 12, 16, 10 5/8, infeasible]

12. Consider the following problem.

Maximize $z = 3x_1 + x_2 + 4x_3$

Subject to :

$$6x_1 + 3x_2 + 5x_3 \le 25$$
$$3x_1 + 4x_2 + 5x_3 \le 20$$
$$x_i \ge 0,\ \ i = 1,2,3.$$

The corresponding final set of equations yielding the optimal solution is

$$\begin{aligned} z \quad + \quad 2x_2 \quad + 1/5\, x_4 + 3/5\, x_5 &= 17 \\ x_1 - 1/3\, x_2 \quad + 1/3\, x_4 - 1/3\, x_5 &= 5/3 \\ x_2 + x_3 - 1/5\, x_4 + 2/5\, x_5 &= 3 \end{aligned}$$

Find the new optimal solution if a new variable x_{new} has been introduced into the model as follow:

(i) Maximize $z = 3x_1 + x_2 + 4x_3 + 2x_{\text{new}}$

Subject to :

$$6x_1 + 3x_2 + 5x_3 + 3x_{\text{new}} \le 25$$
$$3x_1 + 4x_2 + 5x_3 + 2x_{\text{new}} \le 20$$
$$x_i \ge 0,\ \ i = 1,2,3,\ x_{\text{new}} \ge 0.$$

(ii) Maximize $z = 3x_1 + x_2 + 4x_3 + 2\ 1/2 x_{\text{new}}$

Subject to :

$$6x_1 + 3x_2 + 5x_3 + 4x_{\text{new}} \le 25$$
$$3x_1 + 4x_2 + 5x_3 + 3x_{\text{new}} \le 20$$
$$x_i \ge 0,\ \ i = 1,2,3,\ x_{\text{new}} \ge 0.$$

[(i) 0, 0, 2, 5 (ii) 5/3, 0, 3, 0]

13. Consider the following problem:

$$\text{Max. } z = 3x_1 + x_2 + 4x_3$$
st
$$6x_1 + 3x_2 + 5x_3 \leq 25$$
$$3x_1 + 4x_2 + 5x_3 \leq 20$$
$$x_i \geq 0,\ i = 1, 2, 3.$$

The corresponding final set of equations yielding the optimal solution is

$$z + 2x_2 + 1/5x_4 + 3/5x_5 = 17$$
$$x_1 - 1/3x_2 + 1/3x_4 - 1/3x_5 = 5/3$$
$$x_2 + x_3 - 1/5x_4 + 2/5x_5 = 5$$

Identify the optimal solution for the dual problem from the final set of equations.

[(1/5,3/5),z=17]

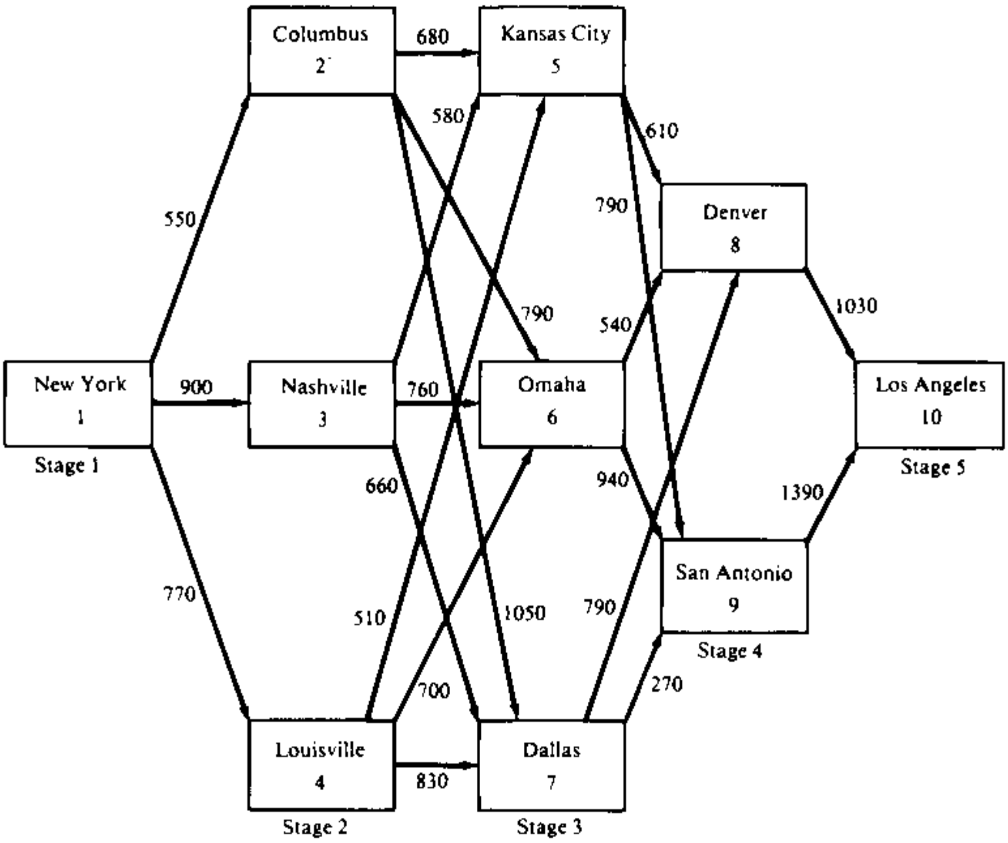
You are known to conduct sensitivity analysis by independently investigating each of the following changes in the original model. For each change, test for feasibility and for optimality (do not reoptimize)

(a) Change the coefficient of x_2 in the objective function to $c_2 = 4$
(b) Change the coefficient of x_3 in the objective function to $c_3 = 3$
(c) Change the coefficient of x_2 in constraint 2 to $a_{22} = 1$
(d) Change the coefficient of x_1 in constraint 1 to $a_{11} = 10$

[not optimal but feasible, optimal & feasible, optimal & feasible, optimal & feasible]

B. Dynamic Programming (DP)

14. JC lives in New York City, but he plans to drive to Los Angeles to seek fame and fortune. JC'c funds are limited , so he has decided to spend each night on his trip at a friend's house. JC has a friend in Columbus, Nashville, Louisville, Kansas City, Omaha, Dallas, San Antonio, and Denver. JC knows that after one day's drive he can reach Columbus, Nashville or Louisville. After two days of driving, he can reach Kansas City, Omaha or Dallas. After three days of driving, he can reach San Antonio, or Denver. Finally, after four days of driving, he can reach Los Angeles. To minimize the number of miles traveled, where should JC spend each night of the trip? The actual mileages between cities are given below. [shortest path:2870 miles]



15. An independent trucker has 8 m^3 of available space on a truck scheduled to depart for New York City. A distributor with large quantities of three different appliances, all destined for New York City, has offered the trucker the following fees to transport as many items as the truck can accommodate:

Appliance	Fee, $ / item	Volume, m^3/item
I	11	1
II	32	3
III	58	5

How many items of each appliance should the trucker accept to maximize shipping fees without exceeding the truck's available capacity? [3,0,1]

16. Suppose that a 10-lb knapsack is to be filled with items listed in table 27-1. To maximize total benefit, how should the knapsack be filled?

[Max:25, one type 1 & two type 2]

Table 27.1

	Weight	Benefit
Item 1	4 lb	11
Item 2	3 lb	7
Item 3	5 lb	12

17. A vending machine company currently operates a 2-year-old machine at a certain location. The following table gives estimates of upkeep, replacement cost, and income (all in dollars) for any machine at this location, as functions of the age of the machine.

	Age, u					
	0	1	2	3	4	5
Income, I(u)	10 000	9 500	9 200	8 500	7 300	6 100
Maintenance, M(u)	100	400	800	2000	2800	3300
Replacement, R(u)	…	3 500	4 200	4 900	5 800	5 900

As a matter of policy, no machine is ever kept past its sixth anniversary and replacement are only with new machines. Determine a replacement policy that will maximize the total profit from this one location over the next 4 years.

[keep-replace-keep-keep]

18. A small construction company currently has a 1-year-old dump truck. Estimates of its upkeep, replacement cost, and the revenues it can be expected to generate, together with similar data for new trucks that may be purchased in the future, are given in the table below; all amounts are in units of $1000. Trucks are never kept more than 3 years, and replacement are only with new models. Determine a maximum-profit replacement policy for this company over the next 5 years.

	Age	Revenue	Upkeep	Replacement
Current Model	1	20	8	18
	2	17	11	25
	3	…	…	35
New Model	0	21	1	…
	1	20	8	19
	2	17	11	26
	3	…	…	36
Next Year's Model	0	21	1	…
	1	17	7	18
	2	15	12	26
	3	…	…	36
Model Two Year Hence	0	22	2	…
	1	19	8	19
	2	17	12	24
	3	…	…	37
Model Three Years Hence	0	24	3	…
	1	18	4	12
	2	15	11	27
	3	…	…	37
Model Four Years Hence	0	25	3	…
	1	19	5	13
	2	14	10	27
	3	…	…	38

19. Ali lives in City 1. He owns insurance agencies in City 2, 3 and 4. Each month, he visits each of his insurance agencies. The distance between each of his agencies in km is shown as follow:

City	1	2	3	4
1	-	525	650	303
2	525	-	535	621
3	650	535	-	415
4	303	621	415	-

What order of visiting his agencies will minimize the total distance traveled?

[1-2-3-4-1, 1-4-3-2-1]

20. Solve the following by using DP.

$$\begin{aligned} \text{Maximize } z = \ & 3x_1 + 5x_2 \\ \text{s.t.:} \quad & x_1 \le 4 \\ & 2x_2 \le 12 \\ & 3x_1 + 2x_2 \le 18 \\ & x_i \ge 0,\ i = 1,2. \end{aligned}$$

[2,6]

C. Nonlinear Programming (Lagrange Multipliers)

21. Solve the following:

(a) Min $f = 2x^2 + y^2 - xy - 8x - 3y$
s.t.: $3x + y = 10$

[69/28, 73/28]

(b) Min $z = 3x_1 - x_2 - 3x_3$
s.t.: $x_1^2 + 2x_3^2 = 1$
$x_1 + x_2 - x_3 = 0$

[–2/√6, 3/√6 , 1/√6]

22. A soft drink company has divided Bloomington into two territories. If x_1 dollars are spent on promotion in territory 1, then $6x_1^{1/2}$ cases of soft drink can be sold there, and if x_2 dollars are spent on promotion in territory 2,then $4x_2^{1/2}$ cases of soft drink can be sold there. Each case of soft drink sold in territory 1 sells for $10 and incurs $5 in shipping and production cost. Each case of soft drink sold in territory 2 sells for $9 and incurs $4 in shipping and production cost. A total of $100 is available for promotion. How can the soft drink Company maximize profits?

[400/13, 900/13]